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WOMBAT– A tool for mixed model analyses in quantitative genetics by REML

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Abstract: WOMBAT is a software package for quantitative genetic analyses of continuous traits, fitting a linear mixed model; estimates of covariance components and the resulting genetic parameters are obtained by restricted maximum likelihood. A wide range of models, comprising numerous traits, multiple fixed and random effects, selected genetic covariance structures, random regression models and reduced rank estimation are accommodated. WOMBAT employs up-to-date numerical and computational methods. Together with the use of efficient compilers, this generates fast executable programs, suitable for large scale analyses. Use of WOMBAT is illustrated for a bivariate analysis. The package consist of the executable program, available for LINUX and WINDOWS environments, manual and a set of worked example, and can be downloaded free of charge from <http://agbu.une.edu.au/~kmeyer/wombat.html>.

Keywords: Software, variance components, genetic parameters, mixed model, restricted maximum likelihood

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1 INTRODUCTION

2 One of the fundamental tasks in quantitative genetics involves the estimation of (co)variance
3 components, i.e. partitioning of the total, observed variation into its causal components, in par-
4 ticular variation due to genetic effects. This provides the basis for classical estimation of genetic
5 parameters such as heritabilities and genetic correlations as well as, more recently, applications
6 such as mapping of quantitative trait loci.

7 Today, variance component analyses for continuous traits are predominantly based on the
8 mixed model, and use maximum likelihood or related methods of inference. In particular, the
9 so-called residual or restricted maximum likelihood (REML) is widely used for analyses of con-
10 tinuous traits. Impetus for this was provided by applications from livestock improvement. Hence
11 the ‘standard’ linear model, relating observations for an individual to its genotype is commonly
12 referred to as the ‘animal model’. One of the main attractions of the ‘animal model’ for quanti-
13 tative analyses is that it allows information from all types of relatives in complex pedigrees to be
14 utilised simultaneously.

15 Uptake of mixed model analyses by REML has been furthered by the availability of appro-
16 priate software. While implemented in general statistical packages such as SAS, Genstat or R,
17 more specialised packages such as ASREML, VCE and DMU, tailored to quantitative models are
18 widely used. One of the earliest of such packages available was DFREML (Meyer, 1988). This
19 paper describes its successor, named WOMBAT.

20 FEATURES

21 WOMBAT is a single program, written in FORTRAN95. Its main purpose is the estimation of
22 covariance components and the resulting genetic parameters for continuous traits, fitting a linear,
23 mixed model. WOMBAT has been developed with the analysis of data from animal breeding
24 programmes in mind, and accommodates most models commonly fitted for such data. However,
25 it is readily applicable to similar problems in other areas of applied statistics.

26 WOMBAT allows for a wide range of models, accommodating numerous traits, multiple
27 fixed and random effects, selected genetic covariance structures, random regression models and
28 reduced rank estimation. WOMBAT employs up-to-date methods of computation, teamed with
29 the use of efficient, commercial compilers to generate ‘fast’ executable programs. It is well suited

30 to analyses of large data sets fitting relatively simple models.

31 **Models and types of analyses**

32 WOMBAT assumes a mixed, linear model of analysis. Fixed effects accommodated range
33 from simple means to multiple, cross-classified or nested effects and covariables. Similarly, mul-
34 tiple random effects with a choice of covariance structure can be fitted. By default, WOMBAT
35 treats random effects as distributed proportionally to an identity matrix or, if flagged appropri-
36 ately, proportionally to the numerator relationship matrix among individuals. If applicable, in-
37 breeding coefficients are calculated and the inverse of this matrix is set up from a list of pedigree
38 information, for either the animal or a sire model.

39 Other general structures are readily fitted: WOMBAT allows for multiple, user-defined co-
40 variance structures for which the corresponding inverse can be supplied. Examples where this is
41 appropriate include the estimation of sex-linked genetic variances or analyses fitting marker or
42 QTL effects assumed to be distributed proportionally to their ‘Identity by Descent’ matrix.

43 WOMBAT accommodates standard uni- and multivariate analyses as well as analyses of
44 ‘functional’ data, fitting a so-called random regression (RR) model. For the latter, a choice of
45 basis functions is available: Legendre polynomials and uniform B-splines are provided as in-built
46 options. Alternatively, any function chosen and specified by the user can be used.

47 By default, covariance matrices are estimated as unstructured: while constrained to be positive
48 semi-definite, they are assumed to have full rank and, for q traits or RR regression coefficients,
49 represented by $q(q + 1)/2$ parameters. Recently, there has been increasing interest in reduced
50 rank estimation and factor-analytic models to model covariance matrices more parsimoniously.
51 WOMBAT provides the option of modelling a covariance matrix through its leading $r < q$
52 principal components only, reducing the number of parameters to $r(2q - r + 1)/2$.

53 WOMBAT implements a number of special features: Different models of analysis for dif-
54 ferent traits are readily fitted, and analyses combining traits recorded repeatedly with traits with
55 a single record only are feasible. Random effects with the same covariance structure, such as
56 direct and maternal additive genetic effects, can be correlated. Options are provided to let the
57 user determine which levels of fixed effects to be set to zero to account for rank deficiencies in
58 the incidence matrix. This can reduce numerical problems, in particular for analyses with nested

59 effects. Similarly, levels of random, genetic effects can be selected to be treated as fixed. This
60 helps, for instance, in reducing bias when base animals have been subject to selection or allows
61 for genetic groups to be fitted. Furthermore, the user can define additional functions of the co-
62 variance components estimated to be evaluated, together with their approximate sampling errors.
63 Typical examples include the repeatability or the total heritability in analyses fitting maternal
64 genetic effects.

65 **Other tasks performed**

66 In addition to estimation of covariance components, WOMBAT can perform several auxiliary
67 tasks:

- 68 • Simple best linear unbiased prediction (BLUP) for random effects, and generalised least
69 squares estimation of fixed effects fitted.
- 70 • Observations for a given data set can be replaced with simulated data, sampling the ran-
71 dom effects specified in the model of analysis (parameter file) from multivariate normal
72 distributions.
- 73 • Results from analyses of subsets of traits can be pooled into positive semi-definite matrices,
74 using the method of ‘iterative summation of expanded part matrices’ (Mäntysaari, 1999).
75 Optionally, this can be carried out weighing individual analyses differentially.

76 **User interface**

77 The user interface for WOMBAT consists of a parameter file and a number of run-time,
78 command line options.

79 The parameter file supplies all information on the data and pedigree files, and the model of
80 analysis to be fitted. In addition, this file specifies the initial guess (‘starting values’) for the
81 covariance components to be estimated and, where applicable, extra constraints to be applied or
82 genetic parameters to be calculated. It needs to be set up following a set of simple but specific
83 rules. These are described in detail in the ‘User Notes’ accompanying the program.

84 Command line options are available to modify the default behaviour of WOMBAT for spe-
85 cific analyses. These include basic options to specify a continuation run, regulate the level of

86 screen output or to select an auxiliary task to be carried out. In addition, command line options
87 allow the user to choose the strategy used to order equations in the mixed model, to select the
88 algorithm used to search for the maximum of the likelihood function, to modify the convergence
89 criterion or maximum number of iterates allowed, or to enforce a specific parameterisation.

90 **Technical details**

91 WOMBAT incorporates state-of-the-art numerical procedures to ensure both computational
92 efficiency and reliable maximisation of the likelihood function.

93 To reduce memory requirements and thus facilitate analyses of large data sets and models,
94 WOMBAT employs sparse matrix storage and manipulation for the mixed model equations. In
95 addition, most arrays are allocated dynamically to the exact size required for a particular analysis.
96 This minimizes the memory used and, in essence, imposes limits on size of the mixed model
97 which can be analysed only through the amount of memory (RAM) which can be accessed.

98 The order in which equations in the mixed model are processed can have a dramatic impact
99 on the computational requirements, both memory and computing time required, per likelihood
100 evaluation or REML iterate. WOMBAT employs several specialised, public domain routines to
101 achieve orderings of the mixed model equations, which minimise the ‘fill-in’ generated and thus
102 the operation counts of sparse matrix factorisation or inversion. In particular, METIS (Karypis
103 and Kumar, 1998) has been found to be well capable of generating high quality orderings for
104 large scale analyses of data from livestock improvement schemes (Meyer, 2005b).

105 WOMBAT utilises the fast algorithm of Tier (1990) to calculate inbreeding coefficients. It
106 is thus capable of setting up the numerator relationship matrix between individuals for very large
107 pedigrees quickly.

108 The default algorithm used by WOMBAT to locate the maximum of the likelihood function
109 is the so-called ‘average information’ algorithm, developed by Thompson and co-workers (see
110 Thompson *et al.*, 2005). This is implemented using an automatic differentiation (Smith, 1995)
111 rather than sparse matrix inversion of the coefficient matrix in the mixed model equations. To
112 ensure an increase in the likelihood in each iterate, step sizes are scaled if necessary, using the
113 backtracking line search of Dennis and Schnabel (1996) to determine the optimum scale factor. In
114 addition, the average information matrix is forced to be safely positive definite by adding a small

115 term to its diagonal if a small eigenvalue or large condition number is encountered. Together,
116 these measures ensure reliable converge for a wide range of analyses, even for difficult problems
117 involving numerous traits or multiple random effects.

118 In addition, WOMBAT implements both the standard and ‘parameter expanded’ (Liu *et al.*,
119 1998) expectation maximisation algorithm. These algorithms are most useful in the first few
120 iterates of an analysis, in particular when adequate starting values for the covariance components
121 are not available, and scenarios where the average information algorithm fails. They require the
122 sparse matrix inverse (Misztal and Perez-Encisco, 1993) of the coefficient matrix in each iterate
123 and, often, a large number of iterates. Furthermore, WOMBAT incorporates two derivative-free
124 search strategies, ported from DFREML.

125 Approximate, lower bound sampling errors of parameters and covariance components are
126 derived at convergence from the inverse of the average information matrix. For quantities which
127 are not linear functions of the parameters estimated, e.g. heritabilities or correlations, sampling
128 errors are derived by first approximating the function by its first order Taylor series expansion,
129 and then computing the variance of the latter.

130 **Applications**

131 WOMBAT has been released in mid-2006. Hence there are few published analyses using
132 this software so far. WOMBAT has been used to estimate genetic parameters for large data sets
133 on beef cattle. This involved up 14 traits in multi-trait analyses, considering a number of reduced
134 rank and factor-analytic models (Meyer, 2005a, 2007).

135 **EXAMPLE**

136 In the following, the input required and output provided by WOMBAT are illustrated for a
137 simple, bivariate analysis of data from a selection experiment in mice (see Meyer, 1991), fitting a
138 model with three cross-classified fixed effects and additive genetic effects and permanent environ-
139 mental effects of the litter as random effects. The data and files shown are available as Example
140 2 in the suite of worked examples for WOMBAT.

141 **The parameter file**

142 The parameter file for the example is shown in [Figure 1](#). Specific ‘codes’ – shown in capital

143 letters – are employed to identify the kind of information provided. Some information is given
144 on the same line as the ‘code’. Other, more complex information is given in a block comprising
145 several lines. In this case, ‘codes’ are used to mark the beginning and end of a block. As far as
146 possible, ‘codes’ are simple abbreviations of corresponding English key words. For instance:

147 **COM** specifies a COMMENT on the analysis, to be printed on the output files.

148 **ANAL** gives the type of ANALYSIS to be carried out, which is a MULTIVARIATE analysis of 2
149 traits (**MUV 2**) in this case.

150 **PEDS** gives the name of the PEDIGREE file.

151 **DATA** gives the name of the data file and indicates the start of a block of lines describing the
152 layout of the file. For each of the two TRAITS (**TR1** or **TR2**), this block contains one line
153 for each column. The line gives the name by which this column is referred to subsequently
154 and, if the column represents a fixed or random effect fitted, the maximum number of its
155 levels. For additive genetic effects (`animal` in this case), this is determined from the list of
156 pedigree information. Hence a ‘0’ has been given. The block is closed by a line beginning
157 with an **END**.

158 **MODEL** indicates the start of a block which specifies the MODEL of analysis. This block
159 contains one line for each effect fitted and one line for each trait:

160 **FIX** for FIXED effects fitted,

161 **RAN** for RANDOM effects fitted, and

162 **TR** for a TRAIT to be analysed, followed by the trait number.

163 A genetic effect, assumed to have covariance matrix proportional to the NUMERATOR RE-
164 LATIONSHIP MATRIX is flagged by the qualifier **NRM**. Again the block is terminated by
165 an **END** statement.

166 **VAR** followed by the name of a random effect (or ‘residuals’) and the size of the covari-
167 ance matrix, indicates that the following line(s) give the starting values for the respective
168 (co)VARIANCE components. Only the elements of the *upper* triangle of each covariance
169 matrix are given.

170 Summary of pedigree and data structure

171 WOMBAT provides files with basic summary information on the pedigree and data structure
172 encountered. These are written after the initial ‘set-up’ steps. Performing the set-up steps only

173 and inspecting these files can help to ensure that the model of analysis has been specified and
174 interpreted correctly.

175 **Figure 2** shows part of the pedigree summary file for the example. Given are the numbers
176 of animals in the data and without records, the numbers of sires and dams, and parents and
177 grand-parents (not shown) identified. In addition, numbers of inbred animals, mean inbreeding
178 coefficient and characteristics of the inverse of the numerator relationship matrix (or its user-
179 supplied equivalent) are reported (not shown).

180 Summary statistics reported for the data structure (not shown) comprise means, standard de-
181 viations and ranges for traits and covariables fitted, and numbers of records for each pair of traits.
182 Random effects in the model are listed together with the respective numbers of levels found and
183 the covariance structure specified. Correspondingly, fixed effects fitted and the number of levels
184 identified in the data are listed separately for each trait.

185 **Estimates of covariance components**

186 Part of the main results file for the example is shown in **Figure 3**. After some information
187 identifying the particular analysis, such as the comment line from the parameter file and the input
188 files used, the maximum of the REML log likelihood together with the corresponding Akaike
189 (AIC) and Bayesian (BIC) information criteria are given. Not shown here, this is followed by a
190 list of the parameters as estimated, and details of the convergence criteria for the last 3 iterates.

191 Subsequently, estimates of the covariance matrices are reported, beginning with the residual
192 covariances and the other covariance matrices in the order in which the corresponding starting
193 values have been specified in the parameter file. In addition, WOMBAT attempts to calculate a
194 matrix of total, phenotypic variances as the sum of all covariance matrices estimated. For each
195 matrix, its eigenvalues and the proportion of variance explained, and the corresponding correla-
196 tions and variance ratios (i.e. proportions of the total variance) are given. In addition, the covari-
197 ance components and correlations or variance ratios are listed together with their approximate
198 lower bound sampling errors.

199 **Estimates for fixed and predictions for random effects fitted**

200 WOMBAT obtains solutions, i.e estimates for all fixed effects and predictions for all random
201 effects fitted as a by-product, and these are written out to separate files at the end of the analysis.

202 **Figure 4** shows the estimates of fixed effects for the example. For each effect level, listed are
203 the effect name, original level code, running number, number of records for the level, and the
204 ‘raw’ mean. Solutions are given as estimated, setting the first level of the second and following
205 effect to zero to account for rank deficiencies, and transformed so that they sum to zero. As
206 WOMBAT uses an implementation of the average information REML algorithm which does not
207 involve inverting the coefficient matrix of the mixed model equations, standard errors of estimates
208 are not provided automatically. However, options are available to obtain these if required.

209 AVAILABILITY

210 WOMBAT is available to the scientific community free of charge. It can be obtained by
211 downloading from the WOMBAT Home Page:

`http://agbu.une.edu.au/~kmeyer/wombat.html`

212 Material available comprises the program, documentation and a set of examples.

213 **The Program**

214 WOMBAT is provided as a pre-compiled (‘executable’) program. WOMBAT has been de-
215 veloped in a LINUX environment and is intended to be used under such operating system. It has
216 been tested under Fedora Core 2 to 5 and Ubuntu 7.04. LINUX versions available are:

- 217 • A 64-bit version and 32-bit version compiled using the Pathscale compiler.
- 218 • 32-bit versions, ‘generic’ or optimised for a Pentium IV processor, compiled using the
219 Lahey LF95 compiler. These are limited to analyses using no more than 2 Gb of RAM.
- 220 • A 32-bit version, compiled with all debugging and checking options switched on. This is
221 too slow for general use and intended for use with bug reports only.

222 In addition, WINDOWS versions of WOMBAT, compiled using the g95 compiler, are avail-
223 able. These are intended for demonstration and evaluation of the program or relatively small
224 scale analyses, and have thus been restricted to models with less than 150 000 effects (equations)
225 in total. WINDOWS versions available are:

- 226 • A 32-bit version to run under CYGWIN

- A 32-bit version, compiled under the MinGW, to run in an **MSYS** or **MsDOS** window.

228 **Documentation**

229 A manual ('User notes') for WOMBAT is available as a .pdf file. Currently, the manual
230 comprises just over 60 pages (A4). It describes the tasks performed, options available, input re-
231 quired and output provided. In addition, an appendix gives some technical details on the methods
232 implemented.

233 It is intended to provide additional comments and documentation related to WOMBAT and
234 more general issues of REML estimation in form of a Wiki, i.e. a set of web pages. This includes
235 information on updates or changes to the program, hints on special types of analyses, and general
236 comments or links. While the main Wiki pages are restricted, there is a freely editable 'open'
237 section available. The Wiki can be accessed through a link from the WOMBAT home page. It
238 represents work in progress; to date contents are limited.

239 **The Examples**

240 A suite of worked examples is provided. Examples comprise data, pedigree and parameter
241 files (input), as well as output files generated by WOMBAT. Currently there are 9 examples,
242 most with multiple sub-analyses. By and large examples represent practical types of analyses
243 common to animal breeding applications. Features illustrated include multivariate analyses, ran-
244 dom regression models, analyses fitting multiple random effects, different models for different
245 traits, and reduced rank estimation.

246 **ACKNOWLEDGMENTS**

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```
COM      Example 2 from DFREML : Bivariate analysis of mouse data
ANAL     MUV 2
PEDS     ../muexped.d
DATA     ../muwex2.d
TR1      traitno 2
TR1      animal 0
TR1      sire
TR1      dam
TR1      generation 4
TR1      sex 2
TR1      littersize 7
TR1      litterno 55
TR1      weight
TR2      traitno 2
TR2      animal 0
TR2      sire
TR2      dam
TR2      generation 4
TR2      sex 2
TR2      littersize 7
TR2      litterno 55
TR2      fintake
END
MODEL
FIX      generation
FIX      sex
FIX      littersize
RAN      animal NRM
RAN      litterno
TR       weight 1
TR       fintake 2
END MOD
VAR      animal 2
4.9 1.0 6.0
VAR      litterno 2
1.5 1.0 3.0
VAR      residual 2
1.7 1.0 12.6
```

Figure 1. Parameter file for example

```

===== Version 15-05-2007 ===== **KM** =====
          Program WOMBAT : Summary of Pedigree Information
=====

Example 2 from DFREML : Bivariate analysis of mouse data

Analysis type       : "MUV 2"
Data file          : "../muwex2.d"
Pedigree file      : "../muexped.d"
Parameter file     : "wombat.par"

No. of animal IDs in data file =           = 284
No. of animal IDs in total   =           = 339

*****Pedigree Structure for random effect :    1 *****
Original no. of animals      =           = 339
No. of animals after pruning =           = 329
... proportion (%) remaining =           = 97.1

No. of levels w/out records  =           = 45
No. of levels with records   =           = 284 100.0%
... 2 record(s)              =           = 284 100.0%

No. of animals w/out offspring =          = 227 69.0%
No. of animals with offspring =          = 102 31.0%
... and records               =          = 57 17.3%

No. of animals with unknown sire =         = 25
No. of animals with unknown dam  =         = 25
No. of animals with both parents unknown =    = 25
No. of animals with records
... and unknown sire            =          = 0
... and unknown dam             =          = 0
... and both parents unknown    =          = 0
No. of sires                    =          = 50
... with progeny in the data    =          = 40
... with records & progeny in data =        = 28
No. of dams                     =          = 52
... with progeny in the data    =          = 42
... with records & progeny in data =        = 29
          intermediate lines deleted
===== end of file =====15-05-2007=====10:29=====

```

Figure 2. Summary of pedigree information for example

```

===== Version 15-05-2007 ===== **KM** =====

          Program WOMBAT : Estimates of covariance components
=====
Example 2 from DFREML : Bivariate analysis of mouse data
Analysis type       : "MUV 2"
Data file          : "../muwex2.d"
Pedigree file     : "../muexped.d"
Parameter file    : "wombat.par"
No. of traits     = 2
No. of records    = 568
No. of parameters = 9
Maximum log L     = -915.885
-1/2 AIC & AICC  = -924.885          -925.046
-1/2 BIC         = -944.263          "Penalty factor" = 3.153

          intermediate lines deleted
***** Estimates of residual covariances *****
          Order of fit = 2
Covariance matrix
  1  1.6233
  2  2.7690  12.479
Eigenvalues of covariance matrix
Value      13.14  0.96
(%)        93.21  6.79
Matrix of correlations and variance ratios
  1  0.1985
  2  0.6152  0.5706
Covariances & correlations with approximate sampling errors
  1 COVS Z 1 1  1.62330  0.959784  vrat  0.198  0.140
  2 COVS Z 1 2  2.76897  1.08633   corr  0.615  0.283
  3 COVS Z 2 2  12.4792  2.34376  vrat  0.571  0.136
***** Estimates for RE 1 "animal" *****
          No. of levels = 329
          Covariance structure = NRM
          Order of fit = 2
Covariance matrix
  1  5.0422
  2 -0.47615  6.3692
Eigenvalues of covariance matrix
Value      6.52  4.89
(%)        57.16  42.84
Matrix of correlations and variance ratios
  1  0.6165
  2 -0.0840  0.2912
Covariances & correlations with approximate sampling errors
  4 COVS A 1 1  5.04224  1.88914  vrat  0.616  0.163
  5 COVS A 1 2 -0.476152  1.99539  corr  -0.084  0.355
  6 COVS A 2 2  6.36916  3.90471  vrat  0.291  0.161
***** Estimates for RE 2 "litterno" *****
          intermediate lines deleted
***** Estimates of phenotypic covariances *****
Covariance matrix
  1  8.1792
  2  1.5299  21.872
          intermediate lines deleted
Covariances & correlations with approximate sampling errors
 10 COVS T 1 1  8.17919  1.26635
 11 COVS T 1 2  1.52990  1.27992   corr  0.114  0.097
 12 COVS T 2 2  21.8720  2.54322
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```

Figure 3. Estimates of covariance components and genetic parameters for example

```

===== Version 15-05-2007 ===== **KM** =====

      Program WOMBAT : GLS solutions for fixed effects
=====

Example 2 from DFREML : Bivariate analysis of mouse data

Fixed effects for trait no.      1      "weight"
  Effect      Orig.code  Level  Solution      SolSum=0      No.recs      Eff.Mean
1 generation      1      1      3.07446      0.250801      93      23.724
1 generation      2      2      1.76119     -1.06247      84      23.063
1 generation      3      3      3.63532      0.811666     107      25.158
  Effect      Orig.code  Level  Solution      SolSum=0      No.recs      Eff.Mean
2 sex            1      1      0.00000     -1.51550     150      22.656
2 sex            2      2      3.03101      1.51550     134      25.650
  Effect      Orig.code  Level  Solution      SolSum=0      No.recs      Eff.Mean
3 littersize     1      1      0.00000      3.85547      11      26.609
3 littersize     2      2     -4.07949     -0.224017     41      23.722
3 littersize     3      3     -3.13151      0.723960     25      24.864
3 littersize     4      4     -5.20582     -1.35035     36      24.028
3 littersize     5      5     -3.71859      0.136875     96      24.265
3 littersize     6      6     -4.57291     -0.717438     45      24.333
3 littersize     7      7     -6.27997     -2.42450     30      21.973

Fixed effects for trait no.      2      "fintake"
  Effect      Orig.code  Level  Solution      SolSum=0      No.recs      Eff.Mean
1 generation      1      1     -13.4289     -1.28830     93      62.884
                                     intermediate lines deleted
3 littersize     7      7      11.9728      4.39440     30      66.433
===== end of file =====15-05-2007=====10:29=====

```

Figure 4. Fixed effects solution for example