

Mildly penalized maximum likelihood estimation of genetic covariances matrices without tuning

Objectives

What?

- Obtain 'better' estimates of genetic parameters

How?

- Reduce sampling variances & 'loss'
 - ↪ penalty on REML likelihood

Why?

- Estimates are closer to true values
 - ↪ more genetic progress

Penalized REML

- Shrink canonical eigenvalues λ_i towards mean $\bar{\lambda}$
- Assume: $\lambda_i \sim \text{Beta}(\alpha, \beta)$

$$\mathcal{P} \propto \sum_i (\alpha - 1) \log(\lambda_i) + (\beta - 1) \log(1 - \lambda_i)$$

$$\alpha = 1 + \bar{\lambda}(N_E - 2) \quad \beta = 1 + (1 - \bar{\lambda})(N_E - 2)$$

- Pen. likelihood: $\log \mathcal{L}_P = \log \mathcal{L} - \mathcal{P}$
- Use $N_E = \alpha + \beta$ to regulate strength of penalty
 - ↪ 'effective sample size'

Criterion

- Loss** measures matrix divergence
 - ↪ Estimates $\hat{\Sigma}$ vs. population values Σ

$$L(\Sigma, \hat{\Sigma}) = \text{tr}(\Sigma^{-1} \hat{\Sigma}) - \log |\Sigma^{-1} \hat{\Sigma}| - q$$

Simulation

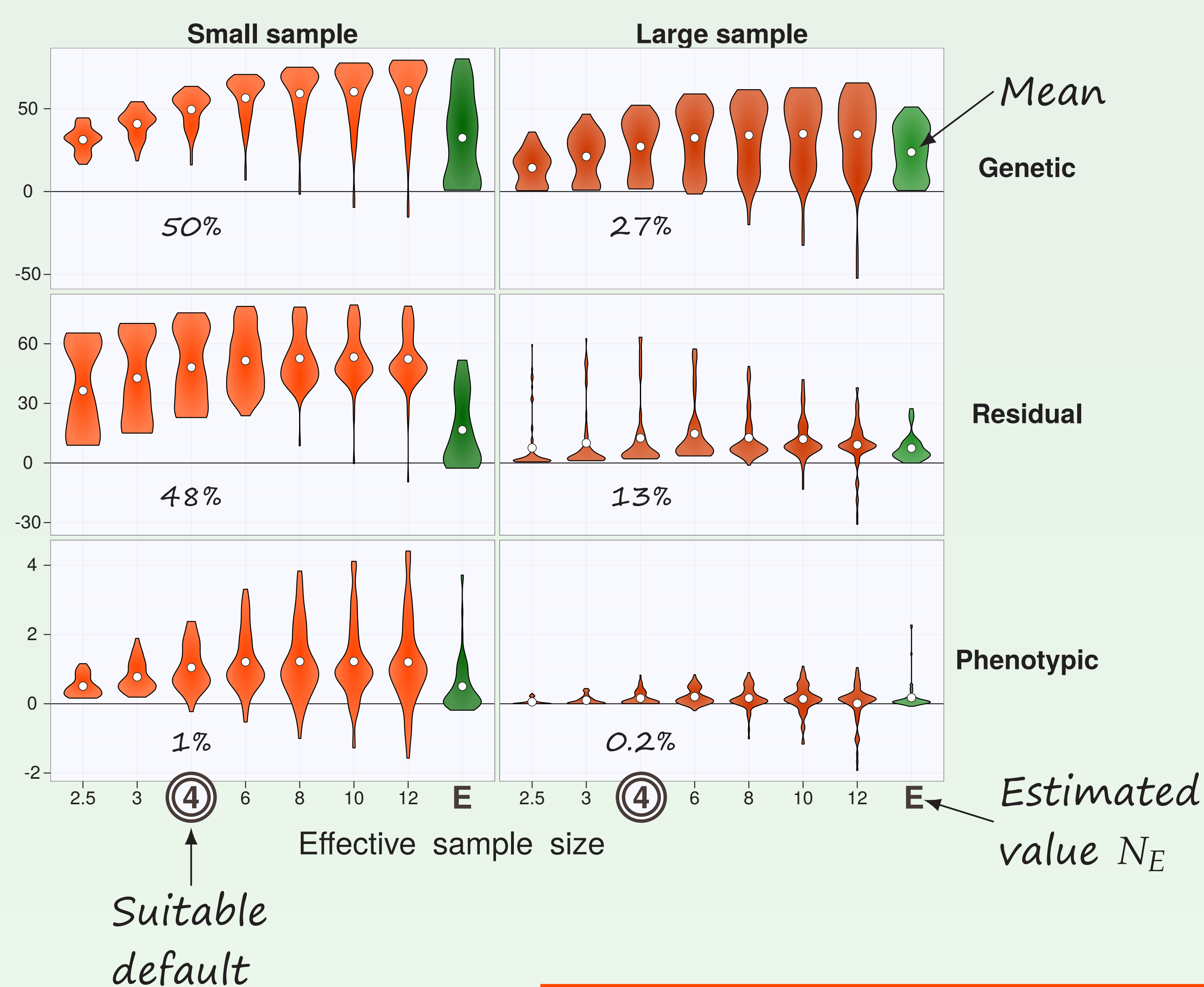
- $q = 9$ traits, 72 sets of pop. values, 500 replicates
- Paternal half-sib design with 10 progeny per sire
 - Small: 100 sires, Large: 1000 sires

Results

- Substantial reductions in loss feasible
 - Mean PRIAL $\approx 50\%$ for small sample
- Spread increases with N_E
 - Low PRIAL if pop. values \neq assumed distribution
 - Negative PRIAL flags too stringent penalization
- Estimation of N_E from data not very successful
 - Laborious & often too low
- Default value $N_E \approx 4$ suited to wide range of pop. values & sample sizes
 - Worthwhile reductions in loss for many cases
 - No detrimental effects

Percentage Reduction In Average Loss (PRIAL) for penalized estimates of covariance matrices

Distribution over 72 sets of pop. values



Conclusions

- Mild penalty on REML estimates of covariance matrices recommended
 - can yield substantial & large proportions of possible reductions in loss
 - can identify adequate defaults for tuning-free application
- No increase in computational complexity \Rightarrow suitable for routine use

Implemented in WOMBAT

